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CFT adapted gauge invariant formulation of arbitrary spin fields in AdS and modified de Donder gauge

R.R. Metsaev*

Department of Theoretical Physics, P.N. Lebedev Physical Institute, Leninsky prospect 53, Moscow 119991, Russia

Abstract

Using Poincaré parametrization of AdS space, we study totally symmetric arbitrary spin massless fields in AdS space of dimension greater than or equal to four. CFT adapted gauge invariant formulation for such fields is developed. Gauge symmetries are realized similarly to the ones of Stueckelberg formulation of massive fields. We demonstrate that the curvature and radial coordinate contributions to the gauge transformation and Lagrangian of the AdS fields can be expressed in terms of ladder operators. Realization of the global AdS symmetries in the conformal algebra basis is obtained. Modified de Donder gauge leading to simple gauge fixed Lagrangian is found. The modified de Donder gauge leads to decoupled equations of motion which can easily be solved in terms of the Bessel function. Interrelations between our approach to the massless AdS fields and the Stueckelberg approach to massive fields in flat space are discussed.

* E-mail: metsaev@lpi.ru

1 Introduction

Further progress in understanding AdS/CFT correspondence [1] requires, among other things, better understanding of field dynamics in AdS space. Conjectured duality of conformal SYM theory and superstring theory in $AdS_5 \times S^5$ has lead to intensive and in-depth study of various aspects of AdS field dynamics. Although many interesting approaches to AdS fields are known in the literature (for review see [2]-[4]), analysis of concrete dynamical aspects of such fields is still a challenging procedure. One of ways to simplify analysis of field and string dynamics in AdS space is based on use of the Poincaré parametrization of AdS space[1]. Use of the Poincaré coordinates simplifies analysis of many aspect of AdS field dynamics and therefore these coordinates have extensively been used for studying the AdS/CFT correspondence. In this paper we develop a formulation which is based on considering of AdS field dynamics in the Poincaré coordinates. This is to say that using the Poincaré parametrization of AdS space we discuss massless totally symmetric arbitrary spin-s, $s \geq 1$, bosonic field propagating in AdS_{d+1} space of dimension $d+1 \geq 4$. Our results can be summarized as follows.

i) Using the Poincaré parametrization of AdS, we obtain gauge invariant Lagrangian for free massless arbitrary spin AdS field. The Lagrangian is *explicitly invariant with respect to boundary Poincaré symmetries*, i.e., manifest symmetries of our Lagrangian are adapted to manifest symmetries of boundary CFT. We show that all the curvature and radial coordinate contributions to our Lagrangian and gauge transformation are entirely expressed in terms of ladder operators that depend on radial coordinate and radial derivative. Besides this, our Lagrangian and gauge transformation are similar to the ones of Stueckelberg formulation of massive field in flat d-dimensional space. General structure of the Lagrangian we obtained is valid for any theory that respects Poincaré symmetries. Various theories are distinguished by appropriate ladder operators.

ii) We find modified de Donder gauge that leads to simple gauge fixed Lagrangian. The surprise is that this gauge gives *decoupled equations of motion*[2]. Note that the standard de Donder gauge leads to coupled equations of motion whose solutions for $s \geq 2$ are not known in closed form so far. In contrast to this, our modified de Donder gauge leads to simple decoupled equations which are easily solved in terms of the Bessel function. Application of our approach to studying the AdS/CFT correspondence may be found in Ref.[10].

Motivation for our study of higher-spin AdS fields in Poincaré parametrization which is beyond the scope of this paper may be found at the end of Section 5.

2 Lagrangian and gauge symmetries

We begin with discussion of field content of our approach. In Ref.[11], the massless spin-s field propagating in AdS_{d+1} space is described by double-traceless $so(d,1)$ algebra totally symmetric tensor field $\Phi^{A_1\ldots A_s}$.[3] This tensor field can be decomposed in scalar, vector, and totally symmetric

[1] Studying $AdS_5 \times S^5$ superstring action [5] in Poincaré parametrization may be found in Ref.[6]. Recent interesting application of Poincaré coordinates to studying $AdS_5 \times S^5$ string $T-$duality may be found in [7] (see also [8]).

[2] Our modified de Donder gauge seems to be unique first-derivative gauge that leads to decoupled equations of motion. Light-cone gauge [9] also leads to decoupled equations of motion, but the light-cone gauge breaks boundary Lorentz symmetries.

[3] $A,B,C = 0,1,\ldots,d$ and $a,b,c = 0,1,\ldots,d-1$ are the respective flat vector indices of the $so(d,1)$ and $so(d-1,1)$ algebras. In Poincaré parametrization of AdS_{d+1} space, $ds^2 = (dx^a dx^a + dzdz)/z^2$. We use the conventions: $\partial_a \equiv \partial/\partial x^a$, $\partial_z \equiv \partial/\partial z$. Vectors of $so(d,1)$ algebra are decomposed as $X^A = (X^a, X^z)$.

tensor fields of the $so(d-1,1)$ algebra:

$$\phi_{s'}^{a_1\ldots a_{s'}}\,, \qquad\qquad s'=0,1,\ldots,s-1,s. \tag{2.1}$$

The fields $\phi_{s'}^{a_1\ldots a_{s'}}$ with $s'>3$ are double-traceless[4]

$$\phi_{s'}^{aabba_5\ldots a_{s'}}=0\,, \qquad\qquad s'=4,5,\ldots,s-1,s. \tag{2.2}$$

The fields in (2.1) subject to constraints (2.2) constitute a field content of our approach. To simplify presentation we use a set of the creation operators α^a, α^z, and the respective set of annihilation operators, $\bar{\alpha}^a$, $\bar{\alpha}^z$. Then, fields (2.1) can be collected into a ket-vector $|\phi\rangle$ defined by[5]

$$|\phi\rangle \equiv \sum_{s'=0}^{s}(\alpha^z)^{s-s'}|\phi_{s'}\rangle\,, \tag{2.3}$$

$$|\phi_{s'}\rangle \equiv \frac{1}{s'!\sqrt{(s-s')!}}\,\alpha^{a_1}\ldots\alpha^{a_{s'}}\,\phi_{s'}^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.4}$$

From (2.3),(2.4), we see that the ket-vector $|\phi\rangle$ is degree-s homogeneous polynomial in the oscillators α^a, α^z, while the ket-vector $|\phi_{s'}\rangle$ is degree-s' homogeneous polynomial in the oscillators α^a, i.e., these ket-vectors satisfy the relations[6]

$$(N_\alpha+N_z-s)|\phi\rangle=0\,, \qquad (N_\alpha-s')|\phi_{s'}\rangle=0\,. \tag{2.5}$$

In terms of the ket-vector $|\phi\rangle$, double-tracelessness constraint (2.2) takes the form[7]

$$(\bar{\alpha}^2)^2|\phi\rangle=0\,. \tag{2.6}$$

Action and Lagrangian we found take the form

$$S=\int d^dxdz\,\mathcal{L}\,, \qquad \mathcal{L}=\frac{1}{2}\langle\phi|E|\phi\rangle\,, \tag{2.7}$$

$\langle\phi|\equiv(|\phi\rangle)^\dagger$, where operator E is given by

$$E \;=\; E_{(2)}+E_{(1)}+E_{(0)}\,, \tag{2.8}$$

$$E_{(2)}\equiv\Box-\alpha\partial\bar{\alpha}\partial+\frac{1}{2}(\alpha\partial)^2\bar{\alpha}^2+\frac{1}{2}\alpha^2(\bar{\alpha}\partial)^2-\frac{1}{2}\alpha^2\Box\bar{\alpha}^2-\frac{1}{4}\alpha^2\alpha\partial\,\bar{\alpha}\partial\bar{\alpha}^2\,, \tag{2.9}$$

$$E_{(1)}\equiv\bar{e}_1\mathcal{A}+e_1\bar{\mathcal{A}}\,, \tag{2.10}$$

$$E_{(0)}\equiv m_1+\alpha^2\bar{\alpha}^2m_2+\bar{m}_3\alpha^2+m_3\bar{\alpha}^2\,, \tag{2.11}$$

$$\mathcal{A}\equiv\alpha\partial-\alpha^2\bar{\alpha}\partial+\frac{1}{4}\alpha^2\,\alpha\partial\,\bar{\alpha}^2\,, \tag{2.12}$$

$$\bar{\mathcal{A}}\equiv\bar{\alpha}\partial-\alpha\partial\bar{\alpha}^2+\frac{1}{4}\alpha^2\,\bar{\alpha}\partial\,\bar{\alpha}^2\,, \tag{2.13}$$

[4] Note that $so(d-1,1)$ tensorial components of the Fronsdal field $\Phi^{A_1\ldots A_s}$ are not double-traceless. Using appropriate transformation (see (5.22)) those tensorial components can be transformed to our fields in (2.1).

[5] We use oscillator formulation [12]-[14] to handle the many indices appearing for tensor fields (see also [15]). It can also be reformulated as an algebra acting on the symmetric-spinor bundle on the manifold M [16].

[6] Throughout this paper we use the following notation for operators constructed out the oscillators and derivatives: $N_\alpha\equiv\alpha^a\bar{\alpha}^a$, $N_z\equiv\alpha^z\bar{\alpha}^z$, $\alpha^2=\alpha^a\alpha^a$, $\bar{\alpha}^2=\bar{\alpha}^a\bar{\alpha}^a$, $\Box=\partial^a\partial^a$, $\alpha\partial=\alpha^a\partial^a$, $\bar{\alpha}\partial=\bar{\alpha}^a\partial^a$.

[7] We adapt the formulation in terms of the double-traceless gauge fields [11]. Adaptation of approach in Ref.[11] to massive fields may be found in Refs.[17, 18]. Discussion of various formulations in terms of unconstrained gauge fields may be found in Refs.[19]-[24]. Study of other interesting approaches which seem to be most suitable for the theory of interacting fields may be found e.g. in Refs.[25]-[27].

$$e_1 = e_{1,1}\Big(\partial_z + \frac{2s+d-5-2N_z}{2z}\Big)\,, \tag{2.14}$$

$$\bar{e}_1 = \Big(\partial_z - \frac{2s+d-5-2N_z}{2z}\Big)\bar{e}_{1,1}\,, \tag{2.15}$$

$$e_{1,1} = \alpha^z f\,, \qquad \bar{e}_{1,1} = f\bar{\alpha}^z\,, \tag{2.16}$$

$$f \equiv \varepsilon\Big(\frac{2s+d-4-N_z}{2s+d-4-2N_z}\Big)^{1/2}\,, \qquad \varepsilon = \pm 1\,, \tag{2.17}$$

$$m_1 = \bar{e}_1 e_1 - 2\frac{2s+d-3-2N_z}{2s+d-4-2N_z} e_1\bar{e}_1\,, \tag{2.18}$$

$$m_2 = -\frac{1}{2}\bar{e}_1 e_1 + \frac{1}{4}\frac{2s+d-2N_z}{2s+d-4-2N_z} e_1\bar{e}_1\,, \tag{2.19}$$

$$m_3 = \frac{1}{2}e_1 e_1\,, \qquad \bar{m}_3 = \frac{1}{2}\bar{e}_1\bar{e}_1\,, \tag{2.20}$$

and subscript n in $E_{(n)}$ (2.8) tells us that $E_{(n)}$ is degree-n homogeneous polynomial in the flat derivative ∂^a. We note that gauge invariance requires $\varepsilon^2 = 1$. Because ε depends on N_z, this leaves two possibilities $\varepsilon = \pm 1$ at least.

The following remarks are in order.

i) Operator $E_{(2)}$ (2.9) is the symmetrized Fronsdal operator represented in terms of the oscillators. This operator does not depend on the radial coordinate and derivative, z, ∂_z, and it takes the same form as the one of massless field in d-dimensional flat space.

ii) Dependence of operator E (2.8) on the radial coordinate and derivative, z, ∂_z, is entirely governed by the operators e_1 and $\bar{e}_1$ which are similar to ladder operators appearing in quantum mechanics. Sometimes, we refer to the operators e_1 and $\bar{e}_1$ as ladder operators[8].

iii) Representation for the Lagrangian in (2.7) -(2.13) is universal and is valid for arbitrary Poincaré invariant theory. Various Poincaré invariant theories are distinguished by ladder operators entering the operator E. This is to say that the operators E of massive and conformal fields in flat space depend on the oscillators α^a, $\bar{\alpha}^a$ and the flat derivative ∂^a in the same way as the operator E of AdS fields (2.8). In other words, the operators E for massless AdS fields, massive and conformal fields in flat space are distinguished only by the operators e_1 and $\bar{e}_1$. For example, all that is required to get the operator E for massive spin-s field in d-dimensional flat space is to make the substitutions

$$e_1 \rightarrow m\alpha^z f\,, \qquad \bar{e}_1 \rightarrow -mf\bar{\alpha}^z\,, \tag{2.21}$$

where m is mass parameter of the massive field and f is given in (2.17). Note also that our field content (2.1) is similar to the one of Stueckelberg formulation of massive field in d-dimensional space [17]. Expressions for e_1, $\bar{e}_1$ appropriate for conformal fields may be found in Refs.[31].

Gauge symmetries. We now discuss gauge symmetries of Lagrangian in (2.7). To this end we introduce the following set of gauge transformation parameters:

$$\xi_{s'}^{a_1\ldots a_{s'}}\,, \qquad s' = 0, 1, \ldots, s-1. \tag{2.22}$$

[8] Interesting application of other ladder operators to studying AdS/QCD correspondence may be found in [28]. We believe that our approach will also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed e.g. in [28]-[30].

The gauge parameters ξ_0, ξ_1^a, and $\xi_{s'}^{a_1\ldots a_{s'}}$, $s'\geq 2$ in (2.22), are the respective scalar, vector, and rank-s' totally symmetric tensor fields of the $so(d-1,1)$ algebra. The gauge parameters $\xi_{s'}^{a_1\ldots a_{s'}}$ with $s'\geq 2$ are subjected to the tracelessness constraint

$$\xi_{s'}^{aaa_3\ldots a_{s'}} = 0\,, \qquad s'\geq 2\,. \tag{2.23}$$

We now, as usually, collect gauge transformation parameters in ket-vector $|\xi\rangle$ defined by

$$|\xi\rangle \equiv \sum_{s'=0}^{s-1} (\alpha^z)^{s-1-s'}|\xi_{s'}\rangle\,, \tag{2.24}$$

$$|\xi_{s'}\rangle \equiv \frac{1}{s'!\sqrt{(s-1-s')!}}\,\alpha^{a_1}\ldots\alpha^{a_{s'}}\,\xi_{s'}^{a_1\ldots a_{s'}}|0\rangle\,. \tag{2.25}$$

The ket-vectors $|\xi\rangle$, $|\xi_{s'}\rangle$ satisfy the algebraic constraints

$$(N_\alpha + N_z - s+1)|\xi\rangle = 0\,, \qquad (N_\alpha - s')|\xi_{s'}\rangle = 0\,, \tag{2.26}$$

which tell us that $|\xi\rangle$ is a degree-$(s-1)$ homogeneous polynomial in the oscillators α^a, α^z, while $|\xi_{s'}\rangle$ is degree-s' homogeneous polynomial in the oscillators α^a. In terms of the ket-vector $|\xi\rangle$, tracelessness constraint (2.23) takes the form

$$\bar\alpha^2|\xi\rangle = 0\,. \tag{2.27}$$

Gauge transformation can entirely be written in terms of $|\phi\rangle$ and $|\xi\rangle$. We find the following gauge transformation:

$$\delta|\phi\rangle = (\alpha\partial - e_1 - \frac{1}{2s+d-6-2N_z}\alpha^2\bar e_1)|\xi\rangle\,, \tag{2.28}$$

where e_1, $\bar e_1$ are given in (2.14),(2.15). From (2.28), we see that the flat derivative ∂^a enters only in $\alpha\partial$-term in (2.28), while the radial coordinate and derivative, z, ∂_z, enter only in the operators e_1, $\bar e_1$. Thus, all radial coordinate and derivative contributions to gauge transformation (2.28) are entirely expressed in terms of the ladder operators e_1 and $\bar e_1$[9].

We finish this Section with the following remark. Introducing new mass-like operator

$$\mathcal{M}^2 \equiv -\bar e_1 e_1 + \frac{2s+d-2-2N_z}{2s+d-4-2N_z}e_1\bar e_1\,, \tag{2.29}$$

and using explicit expressions for operators e_1 and $\bar e_1$ (2.14),(2.15) we find

$$\mathcal{M}^2 = -\partial_z^2 + \frac{1}{z^2}(\nu^2-\frac{1}{4})\,, \qquad \nu\equiv s+\frac{d-4}{2}-N_z\,. \tag{2.30}$$

We make sure that the operators $\mathcal{M}^2$, e_1 $\bar e_1$ satisfy the following commutators:

$$[e_1,\mathcal{M}^2] = 0\,, \qquad [\bar e_1,\mathcal{M}^2] = 0\,. \tag{2.31}$$

Because the operators e_1, $\bar e_1$ enter gauge transformation (2.28), relations (2.31) can be considered as requirement for gauge invariance of the operator $\mathcal{M}^2$. Therefore, $\mathcal{M}^2$ in (2.29) can be considered as a definition of gauge invariant mass operator. We note that making substitutions (2.21) in (2.29) gives $\mathcal{M}^2 = m^2$. Thus, we see that our definition of mass operator $\mathcal{M}^2$ (2.29) gives desired result for massive field in flat space and provides interesting generalization of notion of mass operator to the case of massless AdS field (2.30).

[9] Making substitutions (2.21) in (2.8) and (2.28) one can make sure that our Lagrangian and gauge transformation match with those of flat limit of AdS massive field theory in Ref.[17].

3 Global $so(d,2)$ symmetries

Relativistic symmetries of AdS_{d+1} space are described by the $so(d,2)$ algebra. In our approach, the massless spin-s AdS_{d+1} field is described by the set of the $so(d-1,1)$ algebra fields (2.1). Therefore it is reasonable to represent the $so(d,2)$ algebra so that to respect manifest $so(d-1,1)$ symmetries. For application to the AdS/CFT correspondence, most convenient form of the $so(d,2)$ algebra that respects the manifest $so(d-1,1)$ symmetries is provided by nomenclature of the conformal algebra. This is to say that the $so(d,2)$ algebra consists of translation generators P^a, conformal boost generators K^a, dilatation generator D, and generators J^{ab} which span $so(d-1,1)$ algebra. We use the following normalization for commutators of the $so(d,2)$ algebra generators:

$$[D, P^a] = -P^a\,, \qquad\qquad [P^a, J^{bc}] = \eta^{ab}P^c - \eta^{ac}P^b\,, \tag{3.1}$$

$$[D, K^a] = K^a\,, \qquad\qquad [K^a, J^{bc}] = \eta^{ab}K^c - \eta^{ac}K^b\,, \tag{3.2}$$

$$[P^a, K^b] = \eta^{ab}D - J^{ab}\,, \tag{3.3}$$

$$[J^{ab}, J^{ce}] = \eta^{bc}J^{ae} + 3 \text{ terms}\,. \tag{3.4}$$

Requiring $so(d,2)$ symmetries implies that the action is invariant with respect to transformation $\delta_{\hat{G}}|\phi\rangle = \hat{G}|\phi\rangle$, where the realization of $so(d,2)$ algebra generators $\hat{G}$ in terms of differential operators takes the form

$$P^a = \partial^a\,, \qquad J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}\,, \tag{3.5}$$

$$D = x\partial + \Delta\,, \qquad \Delta \equiv z\partial_z + \frac{d-1}{2}\,, \tag{3.6}$$

$$K^a = -\frac{1}{2}x^2\partial^a + x^aD + M^{ab}x^b + R^a\,, \tag{3.7}$$

$x\partial \equiv x^a\partial^a$, $x^2 \equiv x^ax^a$. In (3.5),(3.7), M^{ab} is spin operator of the $so(d-1,1)$ algebra. Commutation relations for M^{ab} and representation of M^{ab} on space of ket-vector $|\phi\rangle$ (2.3) take the form

$$[M^{ab}, M^{ce}] = \eta^{bc}M^{ae} + 3 \text{ terms}\,, \qquad M^{ab} = \alpha^a\bar{\alpha}^b - \alpha^b\bar{\alpha}^a\,. \tag{3.8}$$

Operator R^a appearing in K^a (3.7) is given by

$$R^a \;=\; -z\widetilde{C}^a\bar{e}_{1,1} + ze_{1,1}\bar{\alpha}^a - \frac{1}{2}z^2\partial^a\,, \tag{3.9}$$

$$\widetilde{C}^a \equiv \alpha^a - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar{\alpha}^a\,, \tag{3.10}$$

where $e_{1,1}$, $\bar{e}_{1,1}$ are given in (2.16). We see that realization of Poincaré symmetries on bulk AdS fields (3.5) coincide with realization of Poincaré symmetries on boundary CFT operators. Note that realization of D- and K^a-symmetries on bulk AdS fields (3.6),(3.7) coincides, by module of contributions of operators Δ and R^a, with the realization of D- and K^a-symmetries on boundary CFT operators. Realizations of the $so(d,2)$ algebra on bulk AdS fields and boundary CFT operators are distinguished by Δ and R^a. The realization of the $so(d,2)$ symmetries on bulk AdS fields given in (3.5)-(3.7) turns out to be very convenient for studying AdS/CFT correspondence [10].

4 Modified de Donder gauge

We begin with discussion of gauge-fixing procedure at the level of Lagrangian (2.7). We find that use of the following *modified de Donder* gauge-fixing term

$$\mathcal{L}_{g.fix} = \frac{1}{2}\langle\phi|E_{g.fix}|\phi\rangle\,, \qquad E_{g.fix} \equiv C\bar{C}\,, \tag{4.1}$$

$$C \equiv \alpha\partial - \frac{1}{2}\alpha^2\bar{\alpha}\partial - e_1\Pi^{[1,2]} + \frac{1}{2}\bar{e}_1\alpha^2\,, \tag{4.2}$$

$$\bar{C} \equiv \bar{\alpha}\partial - \frac{1}{2}\alpha\partial\bar{\alpha}^2 + \frac{1}{2}e_1\bar{\alpha}^2 - \bar{e}_1\Pi^{[1,2]}\,, \tag{4.3}$$

$$\Pi^{[1,2]} \equiv 1 - \alpha^2\frac{1}{2(2N_\alpha + d)}\bar{\alpha}^2\,, \tag{4.4}$$

leads to the surprisingly simple gauge fixed Lagrangian $\mathcal{L}_{total}$:

$$\mathcal{L}_{total} \equiv \mathcal{L} + \mathcal{L}_{g.fix}\,, \tag{4.5}$$

$$\mathcal{L}_{total} = \frac{1}{2}\langle\phi|E_{total}|\phi\rangle\,, \tag{4.6}$$

$$E_{total} = (1 - \frac{1}{4}\alpha^2\bar{\alpha}^2)(\Box - \mathcal{M}^2)\,, \tag{4.7}$$

where $\mathcal{M}^2$ is given in (2.30).[10] We note that our gauge-fixing term (4.1) respects the Poincaré and dilatation symmetries but breaks the conformal boost K^a-symmetries, i.e., the simple form of gauge fixed Lagrangian (4.5) is achieved at the cost of the K^a-symmetries.

We now discuss gauge-fixing procedure at the level of equations of motion. To this end we note that gauge invariant Lagrangian (2.7) leads to the following equations of motion:[11]

$$\Big(E_{(2)} + \Pi^{[2,3]}(E_{(1)} + E_{(0)})\Big)|\phi\rangle = 0\,, \tag{4.8}$$

$$\Pi^{[2,3]} \equiv 1 - (\alpha^2)^2\frac{1}{8(2N_\alpha + d)(2N_\alpha + d + 2)}(\bar{\alpha}^2)^2\,. \tag{4.9}$$

These equations can be represented as

$$(\mathcal{E}_{(2)} + \mathcal{E}_{(1)} + \mathcal{E}_{(0)})|\phi\rangle = 0\,, \tag{4.10}$$

$$\mathcal{E}_{(2)} \equiv \Box - \alpha\partial\bar{\alpha}\partial + \frac{1}{2}(\alpha\partial)^2\bar{\alpha}^2\,, \tag{4.11}$$

$$\begin{aligned}\mathcal{E}_{(1)} &\equiv e_1(\bar{\alpha}\partial - \alpha\partial\bar{\alpha}^2)\\ &+ \bar{e}_1\Big(\alpha\partial + \alpha^2\frac{1}{2N_\alpha + d - 2}\bar{\alpha}\partial - \alpha^2\alpha\partial\frac{1}{2N_\alpha + d}\bar{\alpha}^2\Big)\,,\end{aligned} \tag{4.12}$$

$$\mathcal{E}_{(0)} = m_1 - (m_1 + 4m_2)\alpha^2\frac{1}{2(2N_\alpha + d - 2)}\bar{\alpha}^2 + m_3\bar{\alpha}^2 - \bar{m}_3\alpha^2\frac{2}{2N_\alpha + d - 2}\Pi^{[1,2]}\,. \tag{4.13}$$

[10] Making substitutions (2.21) in (4.2),(4.3) gives gauge fixed Lagrangian for massive field of the form (4.5)-(4.7) with $\mathcal{M}^2 = m^2$.

[11] Appearance of the projector $\Pi^{[2,3]}$ in equations of motion (4.8) is related to the fact that the operators $E_{(1)}$, $E_{(0)}$, in contrast to the symmetrized Fronsdal operator $E_{(2)}$, do not respect double tracelessness constraint (2.6). Note that the ket-vectors $E_{(1)}|\phi\rangle$, $E_{(0)}|\phi\rangle$ are triple-traceless, $(\bar{\alpha}^2)^3E_{(1)}|\phi\rangle = 0$, $(\bar{\alpha}^2)^3E_{(0)}|\phi\rangle = 0$.

Modified de Donder gauge condition is then defined to be

$$\bar{C}|\phi\rangle = 0\,, \tag{4.14}$$

where the operator $\bar{C}$ is given in (4.3). Because of double-tracelessness of $|\phi\rangle$ (2.6), operator $\bar{C}$ (4.3) satisfies the relation $\bar{\alpha}^2\bar{C}|\phi\rangle = 0$, i.e., gauge condition (4.14) respects constraint for gauge transformation parameter $|\xi\rangle$, (2.27). Using the modified de Donder gauge condition in gauge invariant equations of motion (4.10) leads to the following gauge fixed equations of motion:

$$(\Box - \mathcal{M}^2)|\phi\rangle = 0\,, \tag{4.15}$$

where $\mathcal{M}^2$ is defined in (2.30). In terms of fields (2.1), equation (4.15) can be represented as

$$\Big(\Box + \partial_z^2 - \frac{1}{z^2}(\nu_{s'}^2 - \frac{1}{4})\Big)\phi_{s'}^{a_1\ldots a_{s'}} = 0\,, \qquad\qquad \nu_{s'} \equiv s' + \frac{d-4}{2}\,, \tag{4.16}$$

$s' = 0,1,\ldots,s$. Thus, our *modified de Donder gauge condition (4.14) leads to decoupled equations of motion* (4.16) which can easily be solved in terms of the Bessel function [12]. For spin-1 field, gauge condition (4.14), found in [9], turns out to be a modification of the Lorentz gauge.

We note that equations of motion (4.15) have on-shell leftover gauge symmetries. These on-shell leftover gauge symmetries can simply be obtained from generic gauge symmetries (2.28) by the substituting $|\xi\rangle \rightarrow |\xi_{lfov}\rangle$, where the $|\xi_{lfov}\rangle$ satisfies the following equations of motion:

$$(\Box - \mathcal{M}^2)|\xi_{lfov}\rangle = 0\,. \tag{4.17}$$

5 Comparison of standard and modified de Donder gauges

Our approach to the massless spin-s field in AdS_{d+1} is based on use of double-traceless $so(d-1,1)$ algebra fields (2.1). One of popular approaches to the massless spin-s field in AdS_{d+1} is based on use of double-traceless $so(d,1)$ algebra field $\Phi^{A_1\ldots A_s}$ [11]. The aim of this Section is twofold. First we explain how our modified de Donder gauge is represented in terms of the commonly used field $\Phi^{A_1\ldots A_s}$. Also, we compare the modified de Donder gauge and commonly used standard de Donder gauge[13]. Second we show explicitly how our fields (2.1) are related to the field $\Phi^{A_1\ldots A_s}$.

We begin with discussion of modified de Donder gauge-fixing procedure at the level of Lagrangian. First we present gauge invariant Lagrangian for the field $\Phi^{A_1\ldots A_s}$. To simplify presentation we introduce, as before, the following ket-vector

$$|\Phi\rangle = \frac{1}{s!}\Phi^{A_1\ldots A_s}\alpha^{A_1}\ldots\alpha^{A_s}|0\rangle\,, \tag{5.1}$$

$$(\bar{\boldsymbol{\alpha}}^2)^2|\Phi\rangle = 0, \tag{5.2}$$

$$\boldsymbol{\alpha}^2 \equiv \alpha^A\alpha^A\,, \qquad \bar{\boldsymbol{\alpha}}^2 \equiv \bar{\alpha}^A\bar{\alpha}^A\,, \tag{5.3}$$

where (5.2) tells us that the $\Phi^{A_1\ldots A_s}$ is double-traceless, and the scalar products like $\alpha^A\alpha^A$ are decomposed as $\alpha^A\alpha^A = \alpha^a\alpha^a + \alpha^z\alpha^z$. In terms of $|\Phi\rangle$, gauge invariant Lagrangian takes the

[12] Interesting method of solving AdS field equations of motion which is based on star algebra products in auxiliary spinor variables is discussed in Ref.[32].

[13] Recent applications of the *standard* de Donder gauge to the various problems of higher-spin fields may be found in Refs.[33, 34].

form[14]

$$\mathcal{L} = \frac{1}{2}e\langle\Phi|(1-\frac{1}{4}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2)\mathcal{E}|\Phi\rangle\,, \tag{5.4}$$

$$\mathcal{E} \equiv \Box_{AdS} - \boldsymbol{\alpha}\mathbf{D}\bar{\boldsymbol{\alpha}}\mathbf{D} + \frac{1}{2}(\boldsymbol{\alpha}\mathbf{D})^2\bar{\boldsymbol{\alpha}}^2 - s(s+d-5) + 2d - 4 - \boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2\,, \tag{5.5}$$

$$\Box_{AdS} \equiv D^A D^A + \omega^{AAB}D^B\,, \qquad \bar{\boldsymbol{\alpha}}\mathbf{D} \equiv \bar{\alpha}^A D^A\,, \qquad \boldsymbol{\alpha}\mathbf{D} \equiv \alpha^A D^A\,, \tag{5.6}$$

where $e = \det e_\mu^A$, e_μ^A stands for vielbein of AdS_{d+1} space, and D^A are covariant derivatives (for details of notation, see Appendix). Lagrangian (5.4) can be represented as

$$\mathcal{L} \;=\; \frac{1}{2}e\langle\Phi|E|\Phi\rangle\,, \tag{5.7}$$

$$\begin{aligned} E \;\equiv\;& \Box_{AdS} - \boldsymbol{\alpha}\mathbf{D}\bar{\boldsymbol{\alpha}}\mathbf{D} + \frac{1}{2}(\boldsymbol{\alpha}\mathbf{D})^2\bar{\boldsymbol{\alpha}}^2 + \frac{1}{2}\boldsymbol{\alpha}^2(\bar{\boldsymbol{\alpha}}\mathbf{D})^2 - \frac{1}{2}\boldsymbol{\alpha}^2\Box_{AdS}\bar{\boldsymbol{\alpha}}^2 - \frac{1}{4}\boldsymbol{\alpha}^2\boldsymbol{\alpha}\mathbf{D}\,\bar{\boldsymbol{\alpha}}\mathbf{D}\bar{\boldsymbol{\alpha}}^2 \\ -\;& s(s+d-5) + 2d - 4 + \frac{1}{2}(s(s+d-3)-d)\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2\,. \end{aligned} \tag{5.8}$$

We now ready to discuss the modified de Donder gauge. To make our study more useful we discuss both the modified and standard de Donder gauges. Note that our formulas for standard de Donder gauge are valid for arbitrary parametrization of AdS, while the ones for modified de Donder gauge are adapted to the Poincaré parametrization. Gauge-fixing term is defined to be

$$\mathcal{L}_{g.fix} = \frac{1}{2}e\langle\Phi|E_{g.fix}|\Phi\rangle\,, \tag{5.9}$$

where operator $E_{g.fix}$ corresponding to the standard de Donder gauge fixing and the modified de Donder gauge fixing is given by

$$E_{g.fix} = \begin{cases} \mathbf{C}_{stand}\bar{\mathbf{C}}_{stand}\,, & \text{standard gauge;} \\ \mathbf{C}_{mod}\bar{\mathbf{C}}_{mod}\,, & \text{modified gauge,} \end{cases} \tag{5.10}$$

and we use the notation

$$\mathbf{C}_{stand} \equiv \boldsymbol{\alpha}\mathbf{D} - \frac{1}{2}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}\mathbf{D}\,, \qquad \bar{\mathbf{C}}_{stand} \equiv \bar{\boldsymbol{\alpha}}\mathbf{D} - \frac{1}{2}\boldsymbol{\alpha}\mathbf{D}\bar{\boldsymbol{\alpha}}^2\,, \tag{5.11}$$

$$\mathbf{C}_{mod} \equiv \mathbf{C}_{stand} - 2\mathbf{C}_\perp^z\,, \qquad \bar{\mathbf{C}}_{mod} \equiv \bar{\mathbf{C}}_{stand} + 2\bar{\mathbf{C}}_\perp^z\,, \tag{5.12}$$

$$\mathbf{C}_\perp^z \equiv \alpha^z - \frac{1}{2}\boldsymbol{\alpha}^2\bar{\alpha}^z\,, \qquad \bar{\mathbf{C}}_\perp^z \equiv \bar{\alpha}^z - \frac{1}{2}\alpha^z\bar{\boldsymbol{\alpha}}^2\,. \tag{5.13}$$

We now make sure that the gauge fixed Lagrangian $\mathcal{L}_{total}$ takes the form

$$\mathcal{L}_{total} \equiv \mathcal{L} + \mathcal{L}_{g.fix}\,, \tag{5.14}$$

$$\mathcal{L}_{total} = \frac{1}{2}e\langle\Phi|(1-\frac{1}{4}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2)\mathcal{E}_{total}|\Phi\rangle\,, \tag{5.15}$$

[14] Since Ref.[11], various approaches to massless totally symmetric AdS fields were developed in the literature (see e.g. [12, 9, 35, 16]). We use setup discussed in Ref.[9]. Formulas in Ref.[11] are adapted to AdS_4 with mostly negative metric tensor, while our formulas are adapted to AdS_{d+1} with mostly positive metric tensor. Taking this into account and plugging $d = 3$ in (5.5) we make sure that our operator $\mathcal{E}$ matches with the operator L_0 in Eq.(2.7) in Ref.[11].

$$
\mathcal{E}_{total} = \left\{ \begin{array}{ll} \Box_{AdS} - s(s+d-5) + 2d - 4 - \boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2 \,, & \text{standard gauge;} \\ \Box_{0\,AdS} - s(s+d-4) + 2d - 4 - \boldsymbol{\alpha}^2\bar{\alpha}_z^2 + (2s+d-5)N_z \,, & \text{modified gauge,} \end{array} \right. \tag{5.16}
$$

where $\Box_{0\,AdS} \equiv z^2(\Box + \partial_z^2) + (1-d)z\partial_z$. Alternatively, the operator $\mathcal{E}_{total}$ corresponding to the modified de Donder gauge in (5.16) can be represented as

$$
\mathcal{E}_{total} = \Box_{0\,AdS} - \alpha^2\bar{\alpha}^z\bar{\alpha}^z - \nu^2 + \frac{d^2}{4} \,, \tag{5.17}
$$

where ν is given in (2.30).

We proceed with discussion of gauge-fixing procedure at the level of equations of motion. To this end we note that gauge invariant Lagrangian (5.4) leads to the following equations of motion:

$$
\mathcal{E}|\Phi\rangle = 0 \,. \tag{5.18}
$$

We now define the standard and modified de Donder gauge conditions as

$$
\bar{\mathbf{C}}_{stand}|\Phi\rangle = 0 \,, \qquad \text{standard de Donder gauge;} \tag{5.19}
$$

$$
\bar{\mathbf{C}}_{mod}|\Phi\rangle = 0 \,, \qquad \text{modified de Donder gauge,} \tag{5.20}
$$

where $\bar{\mathbf{C}}_{stand}$, $\bar{\mathbf{C}}_{mod}$ are given in (5.11),(5.12). Using (5.19),(5.20) in (5.18) we get gauge fixed equations of motion

$$
\mathcal{E}_{total}|\Phi\rangle = 0 \,, \tag{5.21}
$$

where $\mathcal{E}_{total}$ is given in (5.16). We note that, because of $\mathbf{C}_\perp^z$- and $\bar{\mathbf{C}}_\perp^z$-terms, the modified de Donder gauge breaks some of the $so(d,2)$ symmetries. In the conformal algebra nomenclature, these broken symmetries correspond to broken conformal boost K^a-symmetries.

From $\mathcal{E}_{total}$ (5.16), we see that, because of $\boldsymbol{\alpha}^2\bar{\alpha}^z\bar{\alpha}^z$-term, the modified de Donder gauge for $|\Phi\rangle$ does not lead to decoupled equations for the ket-vector $|\Phi\rangle$ when[15] $s \geq 2$. It turns out that in order to obtain decoupled equations of motion we should introduce our set of fields in (2.1). We remind that $|\Phi\rangle$ is a double-traceless field (5.2) of the $so(d,1)$ algebra, while $|\phi\rangle$ describes double-traceless fields (2.6) of the $so(d-1,1)$ algebra. This is to say that to get decoupled equations of motion we have to make transformation from the $so(d,1)$ ket-vector $|\Phi\rangle$ to $so(d-1,1)$ ket-vector $|\phi\rangle$. We find the following transformation from the ket-vector $|\Phi\rangle$ to our ket-vector $|\phi\rangle$:

$$
|\phi\rangle = z^{\frac{1-d}{2}}\mathcal{N}\Pi^{\phi\Phi}|\Phi\rangle \,, \tag{5.22}
$$

$$
\Pi^{\phi\Phi} \equiv \Pi_\alpha^{[1]} + \alpha^2\frac{1}{2(2N_\alpha+d)}\Pi_\alpha^{[1]}(\bar{\alpha}^2 + \frac{2N_\alpha+d}{2N_\alpha+d-2}\bar{\alpha}^z\bar{\alpha}^z) \,, \tag{5.23}
$$

$$
\Pi_\alpha^{[1]} \equiv \Pi^{[1]}(\alpha, 0, N_\alpha, \bar{\alpha}, 0, d) \,, \tag{5.24}
$$

$$
\mathcal{N} \equiv \Big(\frac{2^{N_z}\Gamma(N_\alpha+N_z+\frac{d-3}{2})\Gamma(2N_\alpha+d-3)}{\Gamma(N_\alpha+\frac{d-3}{2})\Gamma(2N_\alpha+N_z+d-3)}\Big)^{1/2} \,, \tag{5.25}
$$

$$
N_\alpha = \alpha^a\bar{\alpha}^a \,, \quad N_z = \alpha^z\bar{\alpha}^z \,, \tag{5.26}
$$

[15] For spin-1 field, gauge condition (5.20) and the corresponding decoupled equations of motion were found in [9].

where Γ is Euler gamma function and operator $\Pi_\alpha^{[1]}$ in (5.24) is obtained from the function

$$\Pi^{[1]}(\alpha,\alpha^z,X,\bar\alpha,\bar\alpha^z,Y)\equiv\sum_{n=0}^\infty(\alpha^2+\alpha^z\alpha^z)^n\frac{(-)^n\Gamma(X+\frac{Y-2}{2}+n)}{4^n n!\Gamma(X+\frac{Y-2}{2}+2n)}(\bar\alpha^2+\bar\alpha^z\bar\alpha^z)^n\,, \qquad (5.27)$$

by equating $\alpha^z=\bar\alpha^z=0$, $X=N_\alpha$, $Y=d$. We introduce the z-factor in r.h.s. of (5.22) to obtain canonically normalized ket-vector $|\phi\rangle$.

Inverse transform of (5.22) takes the form

$$|\Phi\rangle = z^{\frac{d-1}{2}}\Pi^{\Phi\phi}\mathcal{N}|\phi\rangle\,, \qquad (5.28)$$

$$\Pi^{\Phi\phi} \;\equiv\; \Pi^{[1]}_{\boldsymbol\alpha}+\boldsymbol\alpha^2\frac{1}{2(2N_{\boldsymbol\alpha}+d+1)}\Pi^{[1]}_{\boldsymbol\alpha}(\bar\alpha^2-\frac{2}{2N_{\boldsymbol\alpha}+d-1}\bar\alpha^z\bar\alpha^z)\,, \qquad (5.29)$$

$$\Pi^{[1]}_{\boldsymbol\alpha}\equiv\Pi^{[1]}(\alpha,\alpha^z,N_{\boldsymbol\alpha},\bar\alpha,\bar\alpha^z,d+1)\,,\qquad N_{\boldsymbol\alpha}\equiv N_\alpha+N_z\,, \qquad (5.30)$$

where $\boldsymbol\alpha^2$ is given in (5.3) and $\Pi^{[1]}_{\boldsymbol\alpha}$ is obtained from $\Pi^{[1]}$ (5.27) by equating $X=N_{\boldsymbol\alpha}$, $Y=d+1$.

We now ready to compare modified de Donder gauges for $|\phi\rangle$ (4.14) and $|\Phi\rangle$ (5.20). Inserting (5.28) in (5.20) and choosing $\varepsilon=-1$ in (2.17), we make sure that modified de Donder gauge for $|\Phi\rangle$ (5.20) amounts to modified de Donder gauge for $|\phi\rangle$ (4.14) i.e., modified de Donder gauges for $|\phi\rangle$ (4.14) and $|\Phi\rangle$ (5.20) match. Also we make sure that inserting (5.28) in equations (5.21) leads to equations (4.15), i.e., equations of motions for $|\phi\rangle$ and $|\Phi\rangle$ match. Finally, one can make sure that gauge invariant Lagrangian for $|\Phi\rangle$ (5.4) and the one for $|\phi\rangle$ (2.7) match.

We now compare gauge transformation of the ket-vector $|\phi\rangle$ (2.28) and gauge transformation of $|\Phi\rangle$ which takes the form

$$\delta|\Phi\rangle=\boldsymbol\alpha\mathbf{D}|\Xi\rangle\,,\qquad |\Xi\rangle=\frac{1}{(s-1)!}\alpha^{A_1}\ldots\alpha^{A_{s-1}}\Xi^{A_1\ldots A_{s-1}}|0\rangle\,, \qquad (5.31)$$

where gauge transformation parameter $\Xi^{A_1\ldots A_{s-1}}$ is traceless, $\Xi^{AAA_3\ldots A_{s-1}}=0$, i.e., $\bar{\boldsymbol\alpha}^2|\Xi\rangle=0$. To this end we note that gauge transformation parameters $|\xi\rangle$ and $|\Xi\rangle$ are related as

$$|\xi\rangle=z^{\frac{3-d}{2}}\mathcal{N}'\Pi^{[1]}_\alpha|\Xi\rangle\,,\qquad |\Xi\rangle=z^{\frac{d-3}{2}}\Pi^{[1]}_{\boldsymbol\alpha}\mathcal{N}'|\xi\rangle\,, \qquad (5.32)$$

$$\mathcal{N}'\equiv\mathcal{N}|_{N_\alpha\to N_\alpha+1}\,, \qquad (5.33)$$

where $\Pi^{[1]}_\alpha$, $\Pi^{[1]}_{\boldsymbol\alpha}$, $\mathcal{N}$ are given in (5.24),(5.30), (5.25) respectively. We note that $\Pi^{[1]}_\alpha$ and $\Pi^{[1]}_{\boldsymbol\alpha}$ are projectors on traceless ket-vectors, i.e., if $|\phi_{trf}\rangle$ and $|\Phi_{trf}\rangle$ are tracefull ket-vectors, $\bar\alpha^2|\phi_{trf}\rangle\neq0$, $\bar{\boldsymbol\alpha}^2|\Phi_{trf}\rangle\neq0$, then on has the relations $\bar\alpha^2\Pi^{[1]}_\alpha|\phi_{trf}\rangle=0$, $\bar{\boldsymbol\alpha}^2\Pi^{[1]}_{\boldsymbol\alpha}|\Phi_{trf}\rangle=0$. Using (5.28),(5.32), and $\varepsilon=-1$ in (2.17), we make sure that gauge transformations (2.28) and (5.31) match.

Finally we compare realization of $so(d,2)$ symmetries on the ket-vectors $|\phi\rangle$ and $|\Phi\rangle$. To this end we note that on space of $|\Phi\rangle$ realization of the $so(d,2)$ algebra transformations takes the form

$$\delta_{P^a}|\Phi\rangle=\partial^a|\Phi\rangle\,,\qquad \delta_{J^{ab}}|\Phi\rangle=(x^a\partial^b-x^b\partial^a+M^{ab})|\Phi\rangle\,, \qquad (5.34)$$

$$\delta_D|\Phi\rangle=x^B\partial^B|\Phi\rangle\,,\qquad \delta_{K^a}|\Phi\rangle=(-\frac{1}{2}x^Bx^B\partial^a+x^ax^B\partial^B+M^{aB}x^B)|\Phi\rangle\,, \qquad (5.35)$$

where $x^Bx^B=x^bx^b+z^2$, $x^B\partial^B=x^b\partial^b+z\partial_z$, $M^{aB}x^B=M^{ab}x^b-M^{za}z$. Comparing (3.5) and (5.34), we see that the realizations of Poincaré symmetries on $|\phi\rangle$ and $|\Phi\rangle$ match from the very beginning. Taking into account z-factor in (5.28), it is easily seen that D-transformations for $|\phi\rangle$ (3.6)

and $|\Phi\rangle$ (5.35) also match. All that remains to do is to match conformal boost K^a-transformations given in (3.7) and (5.35). Choosing $\varepsilon = -1$ in (2.17), we make sure that realizations of the operator K^a on $|\phi\rangle$ (3.7) and on $|\Phi\rangle$ (5.35) match.

To summarize, using the Poincaré parametrization of AdS space, we have developed the CFT adapted formulation of massless arbitrary spin AdS field. In our approach, Poincaré symmetries of the Lagrangian are manifest. As is well known string theory solutions like $AdS_{d+1}\times S^{d+1}$ and Dp-brane backgrounds supported by RR-charges have the respective the d- and $(p+1)$-dimensional Poincaré symmetries. We note that the structure of the Lagrangian we obtained for AdS field is valid for any theory that respects Poincaré symmetries. Various theories are distinguished by appropriate ladder operators. Therefore we think that our approach might be a good starting point for formulation of higher-spin gauge fields theory in $AdS_{d+1}\times S^{d+1}$ and Dp-brane backgrounds. For the case of AdS_{d+1} field, the ladder operators depend on the radial coordinate and the radial derivative. It would be interesting to unravel a structure and role of ladder operators in $AdS_{d+1}\times S^{d+1}$ and Dp-brane backgrounds[16]. The $AdS_{d+1}\times S^{d+1}$ and Dp-brane backgrounds play important role in studying string/gauge theory dualities. Developing a theory of higher-spin gauge fields in these backgrounds might be useful for better understanding string/gauge theory dualities.

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Appendix A Notation

Vector indices of the $so(d-1,1)$ algebra take the values $a,b,c = 0,1,\ldots,d-1$, while vector indices of the $so(d,1)$ algebra take the values $A,B,C = 0,1,\ldots,d-1,d$. We use mostly positive flat metric tensors η^{ab}, η^{AB}. To simplify our expressions we drop η_{ab}, η_{AB} in the respective scalar products, i.e., we use $X^aY^a \equiv \eta_{ab}X^aY^b$, $X^AY^A \equiv \eta_{AB}X^AY^B$. Using the identification $X^d \equiv X^z$ gives the following decomposition of the $so(d,1)$ algebra vector: $X^A = X^a, X^z$. This implies $X^AY^A = X^aY^a + X^zY^z$.

We use the creation operators α^a, α^z, and the respective annihilation operators $\bar\alpha^a$, $\bar\alpha^z$,

$$[\bar\alpha^a,\alpha^b] = \eta^{ab}\,, \qquad [\bar\alpha^z,\alpha^z] = 1\,, \qquad \bar\alpha^a|0\rangle = 0\,, \qquad \bar\alpha^z|0\rangle = 0\,. \tag{A.1}$$

These operators are referred to as oscillators in this paper. The oscillators α^a, $\bar\alpha^a$ and α^z, $\bar\alpha^z$, transform in the respective vector and scalar representations of the $so(d-1,1)$ algebra and satisfy the hermitian conjugation rules, $\alpha^{a\dagger} = \bar\alpha^a$, $\alpha^{z\dagger} = \bar\alpha^z$. Oscillators α^a, α^z and $\bar\alpha^a$, $\bar\alpha^z$ are collected into the respective $so(d,1)$ algebra oscillators $\alpha^A = \alpha^a,\alpha^z$ and $\bar\alpha^A = \bar\alpha^a,\bar\alpha^z$.

$x^A = x^a, z$ denote coordinates in $d+1$-dimensional AdS_{d+1} space,

$$ds^2 = \frac{1}{z^2}(dx^adx^a + dzdz)\,, \tag{A.2}$$

while $\partial_A = \partial_a, \partial_z$ denote the respective derivatives, $\partial_a \equiv \partial/\partial x^a$, $\partial_z \equiv \partial/\partial z$. We use the notation $\Box = \partial^a\partial^a$, $\alpha\partial = \alpha^a\partial^a$, $\bar\alpha\partial = \bar\alpha^a\partial^a$, $\alpha^2 = \alpha^a\alpha^a$, $\bar\alpha^2 = \bar\alpha^a\bar\alpha^a$.

The covariant derivative D^A is given by $D^A = \eta^{AB}D_B$,

$$D_A \equiv e_A^\mu D_\mu\,, \qquad D_\mu \equiv \partial_\mu + \frac{1}{2}\omega_\mu^{AB}M^{AB}\,, \qquad M^{AB} \equiv \alpha^A\bar\alpha^B - \alpha^B\bar\alpha^A\,, \tag{A.3}$$

[16] It would also be interesting to unravel the ladder operators in the tensionless limit of AdS strings [36, 37]. Also we think that formalism developed in this paper might be useful for the study of $(A)dS$ massive fields [17] and $(A)dS$ partial-massless fields [38]-[41].

$\partial_\mu = \partial/\partial x^\mu$, where e^μ_A is inverse vielbein of AdS_{d+1} space, D_μ is the Lorentz covariant derivative and the base manifold index takes values $\mu = 0, 1, \ldots, d$. The ω_μ^{AB} is the Lorentz connection of AdS_{d+1} space, while M^{AB} is a spin operator of the Lorentz algebra $so(d,1)$. Note that AdS_{d+1} coordinates x^μ carrying the base manifold indices are identified with coordinates x^A carrying the flat vectors indices of the $so(d,1)$ algebra, i.e., we assume $x^\mu = \delta^\mu_A x^A$, where δ^μ_A is Kronecker delta symbol. AdS_{d+1} space contravariant tensor field, $\Phi^{\mu_1\ldots\mu_s}$, is related with field carrying the flat indices, $\Phi^{A_1\ldots A_s}$, in a standard way $\Phi^{A_1\ldots A_s} \equiv e^{A_1}_{\mu_1}\ldots e^{A_s}_{\mu_s}\Phi^{\mu_1\ldots\mu_s}$. Helpful commutators are given by

$$[D^A, D^B] = \Omega^{ABC} D^C - M^{AB}\,, \qquad [\bar{\boldsymbol{\alpha}}\mathbf{D}, \boldsymbol{\alpha}\mathbf{D}] = \Box_{AdS} + \frac{1}{2} M^{AB} M^{AB}\,, \tag{A.4}$$

where $\Omega^{ABC} = -\omega^{ABC} + \omega^{BAC}$ is a contorsion tensor and we define $\omega^{ABC} \equiv e^{A\mu}\omega_\mu^{BC}$.

For the Poincaré parametrization of AdS_{d+1} space, vielbein $e^A = e^A_\mu dx^\mu$ and Lorentz connection, $de^A + \omega^{AB} \wedge e^B = 0$, are given by

$$e^A_\mu = \frac{1}{z}\delta^A_\mu\,, \qquad \omega_\mu^{AB} = \frac{1}{z}(\delta^A_z \delta^B_\mu - \delta^B_z \delta^A_\mu)\,. \tag{A.5}$$

With choice made in (A.5), the covariant derivative takes the form $D^A = z\partial^A + M^{zA}$, $\partial^A = \eta^{AB}\partial_B$.

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